



MATHEMATICS METHODS

ATAR COURSE

FORMULA SHEET

2016

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Measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference,
 r is the radius and D is the diameter
 $A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides

Prism: $V = Ah$, where V is the volume and A is the area of the base

Pyramid: $V = \frac{1}{3} Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
 $V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
 $V = \frac{1}{3} \pi r^2 h$

Sphere: $S = 4\pi r^2$
 $V = \frac{4}{3} \pi r^3$

Exponentials

Index laws: For $a, b > 0$ and m, n real,

$$a^m b^m = (a b)^m \qquad a^m a^n = a^{m+n} \qquad (a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad a^0 = 1$$

$$\text{For } a > 0 \text{ and } m \text{ an integer and } n \text{ a positive integer, } a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Logarithms

For a, b, y, m and n positive real and k real:

$$1 = a^0 \Leftrightarrow \log_a 1 = 0$$

$$y = a^x \Leftrightarrow \log_a y = x$$

$$\log_a mn = \log_a m + \log_a n$$

$$a = a^1 \Leftrightarrow \log_a a = 1$$

$$\log_e x = \ln x$$

$$\log_a (m^k) = k \log_a m$$

Calculus

Differentiation:

$$\text{If } f(x) = y \text{ then } f'(x) = \frac{dy}{dx}$$

$$\text{If } f(x) = \ln x \text{ then } f'(x) = \frac{1}{x}$$

$$\text{If } f(x) = x^n \text{ then } f'(x) = nx^{n-1}$$

$$\text{If } f(x) = \sin x \text{ then } f'(x) = \cos x$$

$$\text{If } f(x) = e^x \text{ then } f'(x) = e^x$$

$$\text{If } f(x) = \cos x \text{ then } f'(x) = -\sin x$$

Product rule:

$$\text{If } y = f(x) g(x)$$

$$\text{or If } y = uv$$

$$\text{then } y' = f'(x) g(x) + f(x) g'(x)$$

$$\text{then } \frac{dy}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}$$

Quotient rule:

$$\text{If } y = \frac{f(x)}{g(x)}$$

$$\text{or If } y = \frac{u}{v}$$

$$\text{then } y' = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$$

$$\text{then } \frac{dy}{dx} = \frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}$$

Chain rule:

$$\text{If } y = f(g(x))$$

$$\text{or If } y = f(u) \text{ and } u = g(x)$$

$$\text{then } y' = f'(g(x)) g'(x)$$

$$\text{then } \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Powers:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

Exponentials:

$$\int e^x dx = e^x + c$$

Natural logarithm:

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\text{and } \int \frac{f'(x)}{f(x)} dx = \ln (f(x)) + c$$

Trigonometry:

$$\int \sin x dx = -\cos x + c$$

$$\text{and } \int \cos x dx = \sin x + c$$

Fundamental

Theorem of Calculus:

$$\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$$

$$\text{and } \int_a^b f'(x) dx = f(b) - f(a)$$

Incremental formula:

$$\delta y \approx \frac{dy}{dx} \delta x$$

Exponential growth and decay:

$$\text{If } \frac{dy}{dt} = ky, \text{ then } y = Ae^{kt}$$

See next page

Random variables, distributions, probability and proportions

Probability: For any event A and its complement \bar{A} , and event B
 $P(A) + P(\bar{A}) = 1$
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $P(A \cap B) = P(A) P(B|A) = P(B) P(A|B)$

In a Bernoulli trial: \bar{x} is the sample proportion \hat{p} ,
 Mean $\mu = p$ and standard deviation $\sigma = \sqrt{p(1-p)}$

In a binomial distribution:
 $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$
 Mean $\mu = np$ and standard deviation $\sigma = \sqrt{np(1-p)}$

Expected value: If X is a discrete random variable,
 $E(x) = \sum p_i x_i$, where x_i are the possible values of X and $p_i = P(X = x_i)$
 If X is a continuous random variable,
 $E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X .

Variance: If X is a discrete random variable,
 $Var(x) = \sum p_i (x_i - \mu)^2$, where $\mu = E(X)$ is the expected value
 If X is a continuous random variable,
 $Var(x) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx$.

A confidence interval for the proportion, p , of a population is:

$$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad , \quad \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$

where \hat{p} is the sample mean,
 n is the sample size and
 z is the cut-off value on the standard normal distribution corresponding to the confidence level.

Margin of error: $E = z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ is the half-width of the confidence interval

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.